Worcester County Mathematics League

Freshman/JV Meet 3
February 8, 2017

COACHES' COPY
ROUNDS, ANSWERS, AND SOLUTIONS

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017

Round 1: Graphing on a Number Line

All answers must be in simplest exact form in the answer section

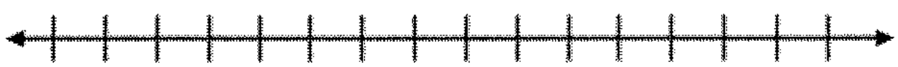
NO CALCULATOR ALLOWED

1. Graph: $-7 \leq 2x-3 < 3$

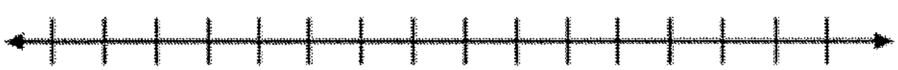
2. Graph: $|2y-14| > 8$

3. Graph: $|2x+5| \leq x+4$

ANSWERS

(1 pt.) 1. 

(2 pts.) 2. 

(3 pts.) 3. 

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017

Round 2: Operations on Polynomials

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find the sum of $5x^2 - 3x - 4$ and $2x^2 - 5$, and then subtract $x^2 + 5x + 7$.

2. Simplify: $(x^3 - 3x^2 + 4)(x^2 + x + 3) - (x^5 - 2x^4 + 5x^2 - 4x + 12)$

3. Expand: $\left[\left(x^{1/2} - \frac{1}{8}\right)^2 + \left(\frac{1}{8} - x^{1/2}\right)^2\right]^2$

ANSWERS

(1 pt.) 1. ______________________________

(2 pts.) 2. ______________________________

(3 pts.) 3. ______________________________

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017

Round 3: Techniques of Counting and Probability

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. The combination to a lock consists of three digits, each of which is between 0 and 9, inclusive. If we know that each digit of the combination is a prime number, how many possible combinations can there be?

2. How many times must you throw a standard pair of dice to ensure that you get the same sum at least two times?

3. For each traveler passing through a security checkpoint, there is a $\frac{1}{n}$ chance of being randomly searched. What is the probability that you and your traveling companion will both pass the checkpoint without being searched?

ANSWERS

(1 pt.) 1. ________________ combinations

(2 pts.) 2. ________________

(3 pts.) 3. ________________

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017

Round 4: Perimeter, Area and Volume

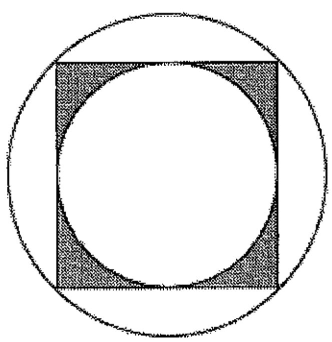
All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find the number of square inches in 6 square feet.

2. A rectangular picture is 3 inches longer than it is wide. The picture is surrounded by a frame which is two inches wide. If the area of the frame is 100 square inches, what is the width of the picture?

3. The figure shows two concentric circles and a square. Find the shaded area if the larger circle has a circumference of 10π units.



ANSWERS

(1 pt.) 1. ____________________ sq inches

(2 pts.) 2. ____________________ inches

(3 pts.) 3. ____________________ sq units

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017

Team Round

All answers must either be in simplest exact form or rounded to EXACTLY three decimal places, unless stated otherwise. (3 points each)

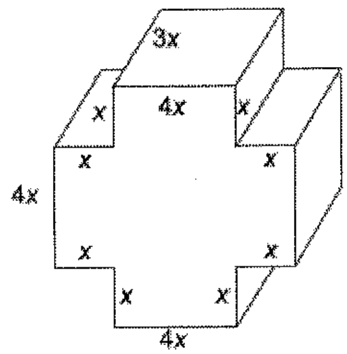
APPROVED CALCULATORS ALLOWED

1. Graph: $[2 \le x+3 \le 5 \;\cup\; 2 \le x-1 \le 4] \;\cap\; \left|x-\frac{5}{2}\right| \le \frac{1}{2}$

2. Simplify: $\left[(2x^3+x^2-25x+12)\div(x-3)\right]\div(2x-1)$

3. A security expert is considering two password systems. System A requires a 3 character password where each character can be a numeral (0 - 9) or a lowercase letter. System B has the same requirements but the password must be either all numerals or all lowercase letters. How many more passwords are possible with System A?

4. Suppose that the surface area of the figure to the right is A square units and that its volume is V cubic units. If $A=\frac{1}{2}V$, find x assuming that $x>0$.



5. Find the sum of all the four digit numbers that can be written using the digits 1, 2, 3 and 4 once each.

6. The force of gravity on the surface of a planet is proportional to M, the mass of the planet, and is inversely proportional to R^2, where R is the radius of the planet. If Planet X has twice the mass of Earth and one third of its radius, how much would a person who weighs 100 pounds on Earth weigh on Planet X?

7. In the figure to the right, the digits 1, 2, 3 and 4 can be used in each row only once and in each column only once. What is the value of x?

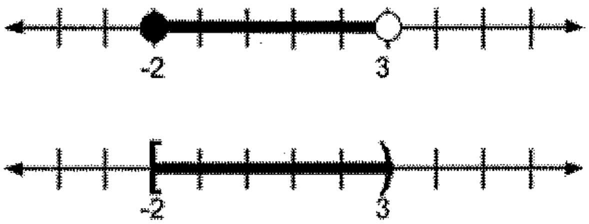
			1
	2		
		x	
1			4

8. What is the smallest composite number greater than or equal to 4 that 291,834,015 is divisible by?

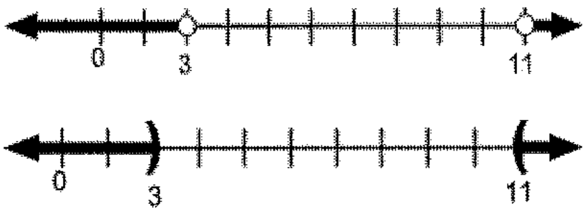
WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017 ANSWER KEY

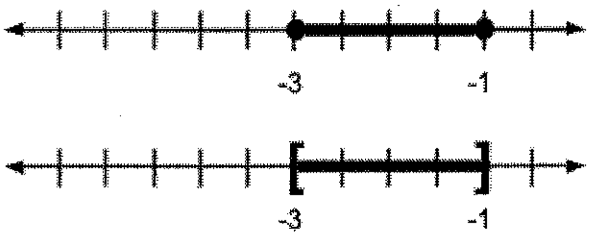
Round 1:

1. 

(Uxbridge)

2. 

(Bartlett)

3. 

(Algonquin)

Round 2:

1. $6x^2 - 8x - 16$ (Shrewsbury)
2. $-10x^2 + 8x$ or $2x(-5x+4)$ (Quaboag)
3. $4x^2 - 2x^{3/2} + \frac{3}{8}x - \frac{1}{32}x^{1/2} + \frac{1}{1024}$

   Alternate acceptable answers are of the form:

   $4x^2 - 2d + ax + be + c$ where

   $a = \frac{3}{8}$ or 0.375 (Doherty)

   $b = -\frac{1}{32}$ or -0.03125 or -2^{-5} or $-\frac{1}{2^5}$

   $c = \frac{1}{1024}$ or 0.000977 or 2^{-10} or $\frac{1}{2^{10}}$

   $d = x^{3/2} = x\sqrt{x} = x^{1.5}$

   $e = x^{1/2} = \sqrt{x} = x^{0.5}$

Round 3:

1. 64 (Doherty)
2. 12 (Westboro)
3. $\left(1-\frac{1}{n}\right)^2$ or $1-\frac{2}{n}+\frac{1}{n^2}$ or $\left(\frac{n-1}{n}\right)^2$ (Tahanto)

Round 4:

1. 864 (St. John's)
2. 9 (St. John's)
3. $50-12.5\pi$ or $50-\frac{25}{2}\pi$ or $50\left(1-\frac{\pi}{4}\right)$ or $50(1-0.25\pi)$ (Southbridge)

TEAM Round

1. 

(Bancroft)

2. $x+4$ (Algonquin)
3. 28080 (Assabet Valley)
4. $2\frac{5}{6}$ or $\frac{17}{6}$ or $2.8\overline{3}$ (Quaboag)
5. 66,660 (Hudson)
6. 1800 (Bromfield)
7. 4 (Auburn)
8. 15 (West Boylston)

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017
Team Round Answer Sheet

1. 

2. ______________________

3. ______________________ passwords

4. ______________________

5. ______________________

6. ______________________ pounds

7. ______________________

8. ______________________

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 3 - February 8, 2017 - SOLUTIONS

Round 1: Graphing on a Number Line

1. Graph: $-7 \leq 2x-3 < 3$

Solution: We have that

$$-7 \leq 2x-3 < 3$$
$$-4 \leq 2x < 6$$
$$-2 \leq x < 3$$

2. Graph: $|2y-14| > 8$

Solution: We have that

$$|2y-14| > 8$$
$$2y-14 > 8 \quad \text{or} \quad 2y-14 < -8$$
$$2y > 22 \quad \text{or} \quad 2y < 6$$
$$y > 11 \quad \text{or} \quad y < 3$$

3. Graph: $|2x+5| \leq x+4$

Solution: We have that

$$|2x+5| \leq x+4$$
$$2x+5 \leq x+4 \qquad \text{and} \qquad 2x+5 \geq -x-4$$
$$x \leq -1 \qquad \text{and} \qquad 3x \geq -9$$
$$x \geq -3$$

Round 2: Operations on Polynomials

1. Find the sum of $5x^2-3x-4$ and $2x^2-5$, and then subtract x^2+5x+7.

Solution: We have that the desired expression is

$$5x^2-3x-4+(2x^2-5)-(x^2+5x+7)$$
$$7x^2-3x-9-(x^2+5x+7)$$
$$6x^2-8x-16$$

2. Simplify: $(x^3-3x^2+4)(x^2+x+3)-(x^5-2x^4+5x^2-4x+12)$

Solution: We have that

$$(x^3-3x^2+4)(x^2+x+3)-(x^5-2x^4+5x^2-4x+12)$$
$$x^5+x^4+3x^3-3x^4-3x^3-9x^2+4x^2+4x+12-(x^5-2x^4+5x^2-4x+12)$$
$$-10x^2+8x$$
$$2x(-5x+4)$$

3. Expand: $\left[\left(x^{1/2}-\tfrac{1}{8}\right)^2+\left(\tfrac{1}{8}-x^{1/2}\right)^2\right]^2$

Solution 1: We have that

$$\left[\left(x^{1/2}-\tfrac{1}{8}\right)^2+\left(\tfrac{1}{8}-x^{1/2}\right)^2\right]^2$$
$$\left[\left(x^{1/2}-\tfrac{1}{8}\right)^2+(-1)^2\left(x^{1/2}-\tfrac{1}{8}\right)^2\right]^2$$
$$\left[2\left(x^{1/2}-\tfrac{1}{8}\right)^2\right]^2$$
$$\left[2\left(x-\tfrac{1}{4}x^{1/2}+\tfrac{1}{64}\right)\right]^2$$

$$4\left[x-\tfrac{1}{4}x^{1/2}+\tfrac{1}{64}\right]\left[x-\tfrac{1}{4}x^{1/2}+\tfrac{1}{64}\right]$$

$$4\left(x^2-\tfrac{1}{4}x^{3/2}+\tfrac{1}{64}x-\tfrac{1}{4}x^{3/2}+\tfrac{1}{16}x-\tfrac{1}{256}x^{1/2}+\tfrac{1}{64}x-\tfrac{1}{256}x^{1/2}+\tfrac{1}{64^2}\right)$$

$$4\left(x^2-\tfrac{2}{4}x^{3/2}+\tfrac{3}{32}x-\tfrac{2}{256}x^{1/2}+\tfrac{1}{64^2}\right)$$

$$4x^2-2x^{3/2}+\tfrac{3}{8}x-\tfrac{1}{32}x^{1/2}+\tfrac{2^2}{2^{12}}$$

Solution 2: Let $a=x^{1/2}$, $b=\frac{1}{8}$ and $c=a-b$ We now have that

$$\left[\left(x^{1/2}-\tfrac{1}{8}\right)^2+\left(\tfrac{1}{8}-x^{1/2}\right)^2\right]^2$$

$$\left[(a-b)^2+(b-a)^2\right]^2$$

$$\left[c^2+(-c)^2\right]^2$$

$$\left[2c^2\right]^2$$

$$4c^4$$

$$4(a-b)^4$$

$$4(a-b)^2(a-b)^2$$

$$4(a^2-2ab+b^2)(a^2-2ab+b^2)$$

$$4(a^4-2a^3b+a^2b^2-2a^3b+4a^2b^2-2ab^3+a^2b^2-2ab^3+b^4)$$

$$4(a^4-4a^3b+6a^2b^2-4ab^3+b^4)$$

$$4\left(x^2-\tfrac{1}{2}x^{3/2}+\tfrac{3}{32}x-\tfrac{1}{128}x^{1/2}+\tfrac{1}{4096}\right)$$

$$4x^2-2x^{3/2}+\tfrac{3}{8}x-\tfrac{1}{32}x^{1/2}+\tfrac{1}{1024}$$

Round 3: Techniques of Counting and Probability

1. The combination to a lock consists of three digits, each of which is between 0 and 9, inclusive. If we know that each digit of the combination is a prime number, how many possible combinations can there be?

Solution: In the digits from 0 to 9, we know that only 2, 3, 5 and 7 are prime numbers. Therefore, since the lock has three digits, we have that there are a total number of $4 \times 4 \times 4 = 64$ combinations.

2. How many times must you throw a standard pair of dice to ensure that you get the same sum at least two times?

Solution: The sums which are possible to throw with a standard pair of dice are the numbers 2 through 12, for 11 possible sums in total.

That means if we throw the pair of dice at least 12 times, we are guaranteed to get the same sum on at least two of the throws. This is an application of the pigeonhole principle, which in this case would state that if 12 items in total are placed in 11 different containers, one container must hold at least 2 items.

3. For each traveler passing through a security checkpoint, there is a $\frac{1}{n}$ chance of being randomly searched. What is the probability that you and your traveling companion will both pass the checkpoint without being searched?

Solution: We have that

Prob(Neither are searched) = Prob(You are not searched) $\times$ Prob(Friend not searched)

Prob(Neither are searched) = $(1-\frac{1}{n}) \times (1-\frac{1}{n})$

Prob(Neither are searched) = $1 - \frac{2}{n} + \frac{1}{n^2}$

Round 4: Perimeter, Area and Volume

1. Find the number of square inches in 6 square feet.

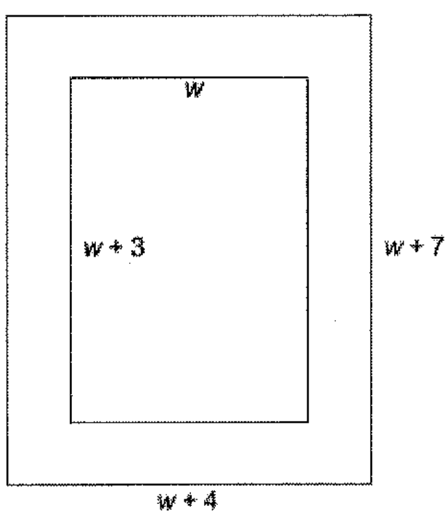
Solution 1 (Dimensional Analysis): We have that there are 12 inches in 1 foot. Therefore,

$$\frac{6\, feet^2}{1} \times \frac{12\ inches}{1\ foot} \times \frac{12\ inches}{1\ foot} = 864\ inches^2$$

Solution 2: We can imagine a rectangle with dimensions 1 foot by 6 feet. This rectangle has an area of precisely 6 square feet. We can convert each dimension into inches to see that the same rectangle is 12 inches by 72 inches. Therefore, the number of square inches in 6 square feet must be equal to $12 \times 72 = 864$.

2. A rectangular picture is 3 inches longer than it is wide. The picture is surrounded by a frame which is two inches wide. If the area of the frame is 100 square inches, what is the width of the picture?

Solution: Let w be the width of the picture. We can draw a diagram of the picture frame.

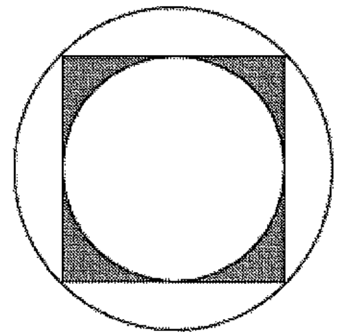


We know that the area of just the picture is simply $w(w+3)$. Therefore, the area of the picture and the frame is precisely $100 + w(w+3)$.

Since we are given that the frame is two inches wide, we know that the width of the frame is $w+4$ and that the length of the frame is $(w+3)+4 = w+7$. Therefore, we know that the area of the picture and the frame is $(w+4)(w+7)$. This fact gives us the following equation:

$$(w+4)(w+7) = 100 + w(w+3)$$
$$w^2 + 11w + 28 = 100 + w^2 + 3w$$
$$8w = 72$$
$$w = 9.$$

3. The figure shows two concentric circles and a square. Find the shaded area if the larger circle has a circumference of 10π units.



Solution: Let R be the radius of the large circle and let r be the radius of the small circle. Notice that the diagonal of the square is equal to $2R$.

Since we are given that the large circle has a circumference of 10π, we have that

$$2\pi R = 10\pi \qquad \Rightarrow \qquad R = 5.$$

Now that we know the value of R, we know that the diagonal of the square is 10. Let x be the length of the square's side. By the Pythagorean Theorem, we know that

$$x^2 + x^2 = 10^2$$
$$2x^2 = 100$$
$$x = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}.$$

Since the small circle is inscribed in the square, we have that

$$2r = x \qquad \Rightarrow \qquad r = \tfrac{5\sqrt{2}}{2}.$$

The shaded area in the figure is equal to the area of the square minus the area of the small circle. This is given by

$$\left(5\sqrt{2}\right)^2 - \pi\left(\tfrac{5\sqrt{2}}{2}\right)^2$$
$$\left(5\sqrt{2}\right)^2 - \tfrac{\pi}{4}\left(5\sqrt{2}\right)^2$$
$$\left(5\sqrt{2}\right)^2\left(1 - \tfrac{\pi}{4}\right) = 50\left(1 - \tfrac{\pi}{4}\right)$$

Team Round

1. Graph: $[2 \le x+3 \le 5 \;\; \cup \;\; 2 \le x-1 \le 4] \;\cap\; \left| x - \frac{5}{2} \right| \le \frac{1}{2}$

Solution: Starting with the left hand term, we have that

$$\begin{array}{lcl} 2 \le x+3 \le 5 & \text{or} & 2 \le x-1 \le 4 \\ -1 \le x \le 2 & \text{or} & 3 \le x \le 5 \end{array}$$

The right hand term gives

$$\left| x - \tfrac{5}{2} \right| \le \tfrac{1}{2}$$
$$-\tfrac{1}{2} \le x - \tfrac{5}{2} \le \tfrac{1}{2}$$
$$2 \le x \le 3$$

Notice that the points 2 and 3 are the only solutions.

2. Simplify: $\left[(2x^3 + x^2 - 25x + 12) \div (x-3)\right] \div (2x-1)$

Solution 1 (Long Division): Beginning with the interior division, we have that

$$\begin{array}{r|l} & 2x^2 + 7x - 4 \\ \hline x-3 & 2x^3 + x^2 - 25x + 12 \\ & \underline{2x^3 - 6x^2} \\ & \quad - 7x^2 - 25x + 12 \\ & \quad \underline{7x^2 - 21x} \\ & \qquad - 4x + 12 \end{array}$$

Then we can perform the exterior division to get that

$$\begin{array}{r|l} & x + 4 \\ \hline 2x-1 & 2x^2 + 7x - 4 \\ & \underline{2x^2 - x} \\ & \qquad 8x - 4 \end{array}$$

Therefore, we have that the expression simplifies to $x+4$.

Solution 2 (Synthetic Division): We can use synthetic division to quickly determine the quotients. Starting with the interior division, we have that

3	2	1	-25	12
	↓	6	21	-12
	2	7	-4	0

This means that the interior quotient is $2x^2+7x-4$. Now we use the same technique to compute the external quotient. To do this, first take out a factor of 2 from the divisor term to get $\left(2x^2+7x-4\right)\div\left[2(x-\tfrac{1}{2})\right]=\left[\left(2x^2+7x-4\right)\div\left(x-\tfrac{1}{2}\right)\right]\div 2$. Now we have that

½	2	7	-4
	↓	1	4
	2	8	0

This leaves us with a quotient of $2x-8$, but we still need to divide by 2 to complete the division. Hence, our final answer is $x-4$.

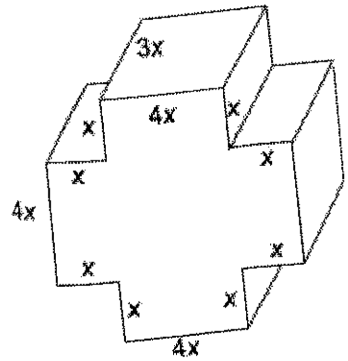
3. A security expert is considering two password systems. System A requires a 3 character password where each character can be a numeral (0 - 9) or a lowercase letter. System B has the same requirements but the password must be either all numerals or all lowercase letters. How many more passwords are possible with System A?

Solution: For System A, there are a total of 36 possible characters that can be used since there are 26 letters and 10 digits. Therefore, the total number of possible passwords with System A is given by $36\times 36\times 36=46,656$

For System B we can either choose to have a password made up only of the 10 digits or a password made up only of the 26 letters. This means that with System B, the total number of possible passwords is given by $(10)^3+(26)^3=18,576$.

Hence, there are $46,656\ -\ 18,576=28,080$ more passwords possible with System A.

4. Suppose that the surface area of the figure to the right is A square units and that its volume is V cubic units. If $A = \frac{1}{2}V$, find x assuming that $x > 0$.



Solution 1: Begin by computing the surface area of the figure. Notice that there are 3 different kinds of faces on the figure: The "plus sign" face on the front and back, the rectangular face with width $4x$ and depth $3x$ on the top, sides and bottom, and finally the rectangular face with width x and depth $3x$, of which there are eight. This gives a surface area of $2(36x^2 - 4x^2) + 4(12x^2) + 8(3x^2) = 136x^2$

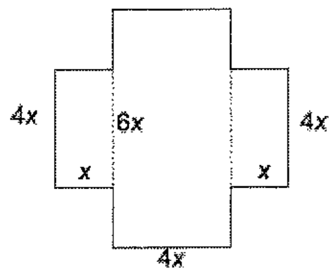
Now compute the volume of the figure. Notice that we can divide the figure into rectangular prisms with height x, width x and depth $3x$. The figure is made up of 32 of these prisms. Since each prism has a volume of $3x^3$, we know that the total volume of the figure is simply $32 \times 3x^3 = 96x^3$.

Now, since we are given that $A = \frac{1}{2}V$, we have that

$$136x^2 = \tfrac{1}{2} \cdot 96x^3$$
$$48x^3 - 136x^2 = 0$$
$$x^2(48x - 136) = 0$$
$$x^2 = 0 \quad \text{or} \quad 48x - 136 = 0$$
$$x = \tfrac{136}{48} = \tfrac{34}{12} = \tfrac{17}{6}$$

Since x must be a positive, we have that $x = \frac{17}{6}$.

Solution 2 (Alternative methods of computing SA and Volume): To compute the surface area of the "plus sign", we can divide the face into 3 boxes as shown to the left:



Adding the areas of these boxes we get $4x^2 + 4x^2 + 24x^2 = 32x^2$.

To compute the volume of the shape quickly, we can simply multiply the area of the "plus sign" by the depth of the shape. This gives $V = 32x^2 \cdot 3x = 96x^3$.

5. Find the sum of all the four digit numbers that can be written using the digits 1, 2, 3 and 4 once each.

Solution: We know that there are $4! = 24$ numbers which can be written using only the digits 1, 2, 3 and 4 once each. It is perfectly feasible to enumerate these 24 numbers and add them up, but there might be a more efficient solution method which is less prone to accidental errors.

Of the 24 numbers we know that 6 must begin with the digit 1, 6 must begin with the digit 2, 6 must begin with the digit 3 and six must begin with the digit 4. Therefore, the sum of the thousands digits of all 24 numbers is simply $6 \times (1+2+3+4) = 6 \times 10 = 60$. (Note: this is 60 thousand)

The same logic applies to the hundreds digits. That is, the sum of the hundreds digits of all 24 numbers must be $6 \times (1+2+3+4) = 6 \times 10 = 60$. (Note: this is 60 hundred = 6 thousand)

Likewise for the tens digits: the sum of the tens digits of all 24 numbers must be $6 \times (1+2+3+4) = 6 \times 10 = 60$. (Note: this is 60 tens = 6 hundred)

Likewise for the units digits: the sum of the tens digits of all 24 numbers must be $6 \times (1+2+3+4) = 6 \times 10 = 60$.

Therefore, the sum of all such numbers is $60,000 + 6,000 + 600 + 60 = 66,660$.

6. The force of gravity on the surface of a planet is proportional to M, the mass of the planet, and is inversely proportional to R^2, where R is the radius of the planet. If Planet X has twice the mass of Earth and one third of its radius, how much would a person who weighs 100 pounds on Earth weigh on Planet X?

Solution: Let M_E and R_E be the mass and radius of the Earth and let M_X and R_X be the mass and radius of planet X.

Since F, the force of gravity, is proportional to M and is inversely proportional to R^2, we know there is a constant k such that $F = k\frac{M}{R^2}$. Therefore, we have that

$$\frac{F_X}{F_E} = \frac{\left(k\frac{M_X}{R_X^2}\right)}{\left(k\frac{M_E}{R_E^2}\right)} = \left(\frac{M_X}{M_E}\right)\left(\frac{R_E}{R_X}\right)^2$$

Since the radius of Earth is 3 times larger than Planet X, we know that $\frac{R_E}{R_X} = 3$ and since the mass of Planet X is twice as large as the mass of Earth, we have $\frac{M_X}{M_E} = 2$. Therefore, we have that

$$\frac{F_X}{F_E} = 2 \times 3^2 \qquad \Rightarrow \qquad F_X = 18F_E$$

Hence, since the force of gravity is 18 times stronger on the surface of Planet X than on the surface of Earth, we have that the person will weigh 1800 pounds on Planet X.

7. In the figure to the right, the digits 1, 2, 3 and 4 can be used in each row only once and in each column only once. What is the value of x?

			1
	2		
		x	
1			4

Solution: Begin by looking at the last row. The digits 2 and 3 need to be in the row in some way; however, we know that 2 cannot be in the second spot since there is already a 2 in that column in the second row. Therefore, we have that the last row must be 1 3 2 4.

			1
	2		
		x	
1	3	2	4

Now we examine the second column. We know that 1 and 4 must be in this column, and yet we also know that there is already a 1 in the first row. Hence, this column must read (from top to bottom)
4 2 1 3.

Now examine the first row. We know that 2 and 3 must be in this row and we also know there is already a 2 in column 3. Therefore, the first row must read 2 4 3 1.

	4		1
	2		
	1	x	
1	3	2	4

2	4	3	1
	2		
	1	x	
1	3	2	4

Finally, examine the third column. We know that there must be a 1 and a 4 in this column, but there is already a 1 in row 3. Therefore, this column must read 3 1 4 2, implying that $x = 4$.

8. What is the smallest composite number greater than or equal to 4 that 291,834,015 is divisible by?

Solution: To solve this problem, we simply need to find the two smallest prime factors of the number in question and then multiply them. The number is not divisible by 2, since it ends with the digit 5. The number is divisible by 3 since $2+9+1+8+3+4+0+1+5=33$ which is divisible by 3. Note that since 33 is not divisible by 9, we know that the full number is not divisible by 9 either.

The number is also divisible by 5 since it ends with the digit 5. Therefore, since 3 and 5 are the number's two smallest prime factors, we know that the smallest composite number that evenly divides the number is $3 \times 5 = 15$.